ANALYSIS

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DEMONSTRATIVES AND PROPER NAMES

By A. M. MacIver

IN this article I am going to try to clear up some confusions regarding demonstrative pronouns and proper names, by reference to the actual use of language for purposes of communication.

Language is used both *symbolically* and *suggestively*. More exactly: every sentence *symbolises* at least some part of its meaning, but in the case of many sentences the remainder of the meaning is only *suggested*. (I speak here of "part of" the meaning of a sentence in such a sense that what is meant by *Something is mortal* would be "part of" what is meant by *Socrates is mortal*.) I say that a sentence "symbolises" a meaning, if that meaning can be understood (by anyone who knows the language in which the sentence is) merely from the composition and structure of the sentence: I say that a sentence "suggests" a meaning, if anyone who is to understand the meaning from the sentence has to take into account factors other than its composition and structure (e.g. the context in which it is uttered). In other words, language is used "suggestively", if the hearer has, to some extent, to *guess* the meaning.

The "composition and structure" that I am here talking about is *grammatical* composition and structure (or, as a philologist would say, more exactly, *lexical and grammatical*)—the fact that the sentence consists of such-and-such words (and not others) grammatically related to one another in such-and-such ways (and not others). And I may say that I take it for granted

that it is only the " type-sentence " (as Ramsey and others call it) which primarily *has* such composition and structure. If we say that a " token-sentence " (so called) has grammatical structure and composition (that it is divided into words, one of which is the grammatical subject, one the main verb, etc. etc.), this is only in a derivative sense : it is only a shorthand way of saying that the " type-sentence " of which it is an instance has this structure and composition. The spoken " token-sentence " may be divided by phoneticians, and the written " token-sentence " by graphologists, into parts within parts *ad infinitum* ; but none of these parts can be called " words ", except in the sense that they represent the words into which the " type-sentence " is divided, of which the " token-sentence " is an instance, (the word being a " part " of the " type-sentence " in the sense of being something which two such sentences, otherwise different, may have in common). This seems to me a fundamental point in the Theory of Language, though it is seldom explicitly recognised. It follows at once from the admission that language, in so far as it is symbolic, is *conventional* : for the convention must connect with the meaning something which can be repeated, which therefore cannot be the particular utterance (" token-sentence "), but must be the " type " of which that utterance is only one instance.

Ordinary language is, to a very large extent, *suggestive*. Anyone who reflects upon an ordinary conversation must observe how much of his meaning the speaker leaves to be guessed by the exercise of the hearer's common sense (i.e. his judgment of what the speaker is *likely* to mean). Speaking in the suburbs of Leeds someone says : *I shall be going to town soon*, and leaves it to his hearer to guess (assuming that he will guess correctly) whether he means that he will be going to the centre of Leeds within the next hour or so, or that he will be going up to London within the next week or so. The *symbolic* use of language is certainly the most interesting ; for by this means alone we are able to communicate a wholly novel meaning (one which nobody could be expected to guess). But the purely symbolic use is almost confined to the sciences and philosophy ; and I doubt whether even here it is more than an ideal limit, to which the scientist or

philosopher strives to approximate (by the use of precisely defined terms, approximation of grammatical to logical form, etc. etc.).

But every sentence (even that which leaves most to suggestion) *symbolises something* : there is something which can be understood merely from the composition and structure of the sentence by anyone who knows the language, or it would not be a sentence at all, but only an ejaculation. For example, the sentence *I shall be going to town soon* would be understood by anyone who knew English, even if he knew nothing whatever about the context (if he met the sentence, say, quoted as an example in an English grammar), to mean at least that someone was supposing that he would be going sometime to some town (and not, say, that there were lions in some zoo).

There are, however, certain linguistic forms which are only used when what is strictly symbolised by the sentence is only a small part of what the speaker wishes to convey, a very important remainder being left to be guessed from the context. (The use of such a form, therefore, constitutes an invitation to make such a guess.) Included among these forms is, I think, the use of demonstrative pronouns and proper names. I believe that the whole confused theory of " logically proper names " is due to the false belief that a complete particular perceptible fact can be *symbolised* in language, when in fact only *part* of it (i.e. a corresponding *general* fact) can be symbolised and the remainder can only be *suggested*.

For example, if, pointing to a conspicuous black-and-white bird with an upturned bill, I say to someone who has never seen such a bird before : *That is an avocet*, all that this sentence strictly symbolises (all that could be understood from it by someone who was unaware of the context of utterance) is that something is an avocet (or in other words, that an avocet exists) ; but I use the demonstrative pronoun *that*, because what I am really concerned to convey to my hearer is not this very indefinite (or general) proposition, but the much more definite one that the bird actually before us is an avocet, which can only be understood from the sentence by someone who recognises and takes into account the context of utterance.

It would be a mistake to say that, in this case, my full meaning is *symbolised*, not indeed by the sentence alone, but by a larger symbol, of which the sentence is one part and my pointing at this particular bird another part. My pointing is simply a device (and my use of the word *that* in conjunction with it a further device) to ensure that my hearer will be attending to this particular bird when he hears the sentence uttered, and thus will be likely to *guess* correctly what I am referring to. If he were already attending to the bird, the pointing could be left out, and so could the use of the word *that*. For example, if we were both engaged in watching a flock of birds, only one of which was black-and-white, I could say simply : *The black-and-white bird is an avocet*, and be tolerably sure that he would guess my reference correctly. I could even say, with the same effect : *There's an avocet feeding a little apart from all the other birds* (where the word *there* is not demonstrative, but used exactly as in *There's milestones on the Dover road*, as is shown by the emphasis), if there was only one bird feeding apart from the others. What is strictly symbolised by *There is an avocet feeding apart from the other birds* is just what might also be symbolised by *There is a flock of birds, one of which is feeding apart from the others and is an avocet*, which is evidently a wholly general proposition : but my hearer judges that I would not be likely to utter it unless it had some relevance to the scene before us, and so he is enabled to identify the avocet.

Among demonstrative pronouns, in this sense, we must include personal pronouns, such as *I* and *you*, and demonstrative adverbs, such as *here* and *there*, *now and then*. For example, the sentence *I am in Leeds* would be properly used by me to convey to a hearer (say, at the other end of the telephone) the proposition that A. M. MacIver was in Leeds at a particular time (namely the time at which I utter the sentence) ; but the sentence does not *symbolise* this proposition, for there is nothing in its structure or composition to indicate that it is about A. M. MacIver at all. The most that is symbolised is the proposition that somebody is not only in Leeds but also saying that he is ; but *who* this is, is left to be guessed by the hearer from evidence outside the symbol itself.

(Tenses of verbs are also demonstratives in the same sense, the use of a tense being equivalent to the use of a demonstrative adverb or adverbial phrase, such as *now* or *before now*.)

I have said that a sentence containing a demonstrative is understood by reference to the *context* of utterance. But there are two *sorts* of contexts, which I will call "*extra-linguistic*" and "*intra-linguistic*" contexts. The extra-linguistic context of an utterance is the external circumstances in which the utterance occurs : in all the examples that I have so far given, the context referred to has been extra-linguistic (the fact that there is only one black-and-white bird in sight, the fact that it is A. M. MacIver who is speaking, etc. etc.). The intra-linguistic context, on the other hand, is the series of sentences uttered in conjunction with that under consideration, or only not uttered because they "go without saying."

The use of a demonstrative in a sentence may suggest a meaning by reference either to an extra-linguistic, or to an intra-linguistic context. For example, if I say : *He was at Harrow and Balliol*, with a slight stress on the word *he* and a glance towards a particular man, I shall be understood, from this extra-linguistic reference, to be talking about that man. If, however, the sentence *He was at Harrow and Balliol* occurs in a narrative part of a novel, the full meaning of the sentence as it there occurs will be understood by a reference back to other sentences which have gone before it in the narrative (or perhaps a reference forward to some which come after), which constitute an intra-linguistic context : there is no reference outside the whole series of the sentences which make up the novel.

But suggestion by reference to an intra-linguistic context (unlike suggestion by reference to an extra-linguistic context) can always be eliminated by restatement. Whatever is represented by a series of sentences could always be represented by a single longer sentence. The choice between the two is purely a matter of literary style. The approved literary style of some languages, such as Latin and German, often uses a single long sentence to express what, in the approved style in English, would be expressed in a series of shorter sentences : but the series of short sentences is *possible* in Latin or German, and the single long

sentence is *possible* in English ; it is only not " good style ". But, if we take a sentence whose full meaning is suggested by reference to an intra-linguistic context and restate the whole series of sentences which form that context, together with the sentence itself, in one long sentence, then the suggestion by intra-linguistic reference will disappear : what was formerly only suggested will now be symbolised. Take for example the series of sentences : *Lord Aubrey de Vere walked down Piccadilly. He carried a walking-stick in his hand.* Consider the second sentence. Only part of its meaning is symbolised, for, from its composition and structure, we can only gather that *somebody* carried a walking-stick in his hand on *some* occasion ; to understand *who* it was and *when*, we must refer to the sentence that went before. But, if we restate these two sentences in one longer sentence, we get *Lord Aubrey de Vere walked down Piccadilly carrying a walking-stick in one hand*, in which this suggestion by intra-linguistic reference has disappeared.

The use of a *proper name* is just like that of a demonstrative, except that it is *only* used to suggest an *intra-linguistic* reference. The simplest case is that of the proper name in fiction. To take W. E. Johnson's example, the fairy-tale begins : *Once upon a time there was a boy whose name was Jack. One day Jack set out to go to market.*—and so proceeds, with a continual use of the name *Jack*. But it would be possible (though inconvenient) to eliminate the proper name, and to begin : *Once upon a time there was a boy, who set out one day to go to market*, and tell the whole story of Jack and the Beanstalk in one long sentence, composed of an immense series of relative clauses, without ever using the name *Jack* at all. The practice would, indeed, soon become intolerably clumsy in English, especially if more than one character was introduced into the story, owing to the uncertainty of reference of the relative pronouns ; but it could be carried a good deal further, using participial constructions as well as relative clauses, in a language (such as Latin or Greek) in which participles decline in case, number and gender, making some of the references more certain ; and, in principle, it could be carried to any extent. The proper name is, in fact, nothing but a stylistic substitute for a relative pronoun. (It follows that the

proper name has only linguistic, not logical, importance.)

In this case the story begins with the existential sentence *Once upon a time there was a boy*, which involves no extra-linguistic reference at all. If it had been history and not fiction, the logical beginning would have been a *demonstrative* sentence; for example: *You see that boy over there? Well, his name is Jack.* —the story continuing, as before, with the name *Jack* used as a substitute for a relative pronoun, referring back to this original sentence. Or the reference might be indirect; for example: *You see that house there? Well, there is a boy who lives in it, whose name is Jack.*—the story proceeding, again, as before. (Or—showing clearly the nature of the demonstrative—the beginning might have been: *You see a house among some trees at the top of a hill? Well, . . .* etc.) Here there *is* extra-linguistic reference, which cannot be eliminated; but it is due, not to the proper name (whose reference is only intra-linguistic), but only to the demonstrative to which the proper name refers back. The whole story of Jack is only a more complicated case of the principle exemplified in *That is an avocet* (being equivalent to *That is a boy who . . .* or *That is a house in which there lives a boy who . . .*) The story, so far as *symbolic* representation goes, might be the story of *any* boy; nobody who merely knew English could tell whose story it was; but the hearer is given cause to *guess* that it is the story of the boy whom he sees, or who lives in the house which he sees.

The demonstrative sentence to which I refer when I use a proper name need not be actually uttered by me, if it is something that "goes without saying". If I want to say something about Mr. Baldwin to a contemporary educated Englishman, I need not begin: *You see this bit of ground that we are now standing on? Well, it is part of a much larger piece of ground, called England, which is all under one government; and the present executive head of that government is a man called Stanley Baldwin.* I can take all that for granted, and begin straight off: *I see that Baldwin has been making a speech.* In the same way, the modern novelist does not begin in the style of the fairy-tale: *Once upon a time there was . . .* He trusts his reader to supply that, and starts straight off: "*Damn,*" *said the Duchess.* But still this is only

understood because the reader *does* supply *Once upon a time there was a Duchess*, as something that the novelist would have asserted if he had thought it necessary to take the trouble. And in the same way, the sentence *I see that Baldwin has been making a speech* is only understood, if there is some demonstrative sentence, which the hearer recognises that the speaker might have uttered if it had been necessary (such as *That is Baldwin*, pointing to the man, or *This is England*, indicating the place), which would suggest (though not, of course, symbolise) something within the hearer's own experience to provide a reference, direct or indirect, for what he was going to say.

University of Leeds, July 1936.

DR. VON JUHOS AND PHYSICALISM

By JUSTUS BUCHLER

TAKING for granted familiarity with Dr. v. Juhos's paper on "Empiricism and Physicalism" (*Analysis*, 2, 6) and Dr. Hempel's reply,[1] I want (I) to remark on a point made by v. Juhos which I think Hempel has somewhat neglected, and (II) to express the relation between v. Juhos's view and that of Physicalism by classifying psychological statements as "significant" and "meaningful".

(I) According to v. Juhos there may be statements about psychological experience ("A has a headache")[2] which are meaningful even though no external manifestation of the experience can be detected. He says: "A statement is only unintelligible if it is logically impossible for everyone to test it. As soon as conditions are imaginable, under which the truth of a statement becomes determinable for everybody, the statement is already generally intelligible, even if the fictitious circumstances

[1] *Some Remarks on Empiricism; Analysis*, 3, 3.

[2] For purposes of clarity I sometimes use *A* (proper name) as subject in the examples given instead of *I* (1st pers.). We are interested in the interpretation of the predicate.

do not really exist." (p. 88.) For the Physicalists, in order to know the meaning of a statement such as " A has a headache," it is not necessary that the feeling should somehow be conveyed to us, but only that we should be able observationally to identify the conditions under which the feeling occurs, and consequently be able to determine the truth or falsity of the statement ; in other words, the truth or falsity of the statement is determined by means of the existence or non-existence of certain specified physical states or overt acts. For v. Juhos, on the other hand, the truth or falsity of the statement " is independent of the behaviourist test " (p. 88), and the statement has meaning because it is " logically possible " for others to have the same experience as that expressed by the statement. Consequently, for v. Juhos, only the subject can verify a statement about his experience ; yet others may know what the statement means, because " conditions are imaginable, under which the truth of the statement becomes determinable for everybody."

What does v. Juhos mean by " determinable for everybody " ? This can be understood in two ways : (1) Determinable for everybody by means of external criteria ; and (2) Determinable for everybody by means of the same experience. V. Juhos means (2). But this still tells us nothing of how B, C, D understand a statement about an experience of A. Consider an example, " A is melancholy ". According to v. Juhos this statement is verifiable only by A, but B, C, D know what it means because it is possible for them also to be melancholy, it is conceivable that they should verify the statement by having the experience it expresses ; that is, in this sense B, C, D know how the truth of the statement is tested. But (*a*) supposing that B, C, D have never had the feeling of melancholy, what does it mean to say that " conditions are imaginable, under which the truth of the statement becomes determinable " for them ? Imaginable for whom ? For B, C, D ? But B, C, D, never having *felt* melancholy, cannot imagine conditions under which they would *feel* melancholy. And surely the conditions are not to be imaginable for E, F, G. For if E, F, G, having felt melancholy, are able to imagine how B, C, D would feel melancholy, this does not help B, C, D to understand what it means for A to

be melancholy. (*b*) Assuming that B, C, D have previously felt melancholy, how do they know, when the statement "A is melancholy" is made, that the feeling which they imagine is not a feeling altogether different from that expressed by the statement? There are no criteria whatever to indicate to them that the word "melancholy" is used in the statement to denote the feeling it denotes for them.

On v. Juhos's theory of the meaning of psychological statements, therefore, the "conditions under which the truth of a statement becomes determinable" are no conditions at all. Under these conditions any statement truthfully expressing some feeling or emotion of the subject, such as "I am noble" or "A feels godlike", would belong to empirical science. On the Physicalist view "A is melancholy" would *mean* for B, C, D that A behaves in a specified way. By means of a psychological law of correlation between melancholy and overt behaviour (of equivalence of statements about psychical states to statements about physical states), they can determine the truth of the statement about melancholy by determining the truth of the statement about behaviour. Thus (*a*) even if B, C, D had never *felt* melancholy they would know what "A is melancholy" *means*; and (*b*) they would always know that for them to be melancholy would mean the same as for A to be melancholy.

(II) We can express the relation of v. Juhos to the Physicalists by the following use of the distinction sometimes made between "significance" and "meaning". Assuming that the statement "A is melancholy" expresses a psychical state and that to this state there corresponds a physical state expressible in a statement equivalent to "A is melancholy", we may say: The statement "A is melancholy" has *significance* for A alone; it has *meaning* for both A and others. It is significant for A alone because only A *feels* melancholy, but it is meaningful for others because they know how to determine whether or not it is true that A feels melancholy (by verifying the physical statement inferrible from "A is melancholy"). The statement becomes significant as well as meaningful for B, C, D when they become melancholy: significant because they have the same feeling, and they assume the feeling to be the same because their physical

state is the same. (If it is doubted that the feeling is " really " the same, then what is doubted is the accuracy of the law of correlation, a matter for psychology to determine.) It is possible that the statement " A is melancholy " should be significant though meaningless. This would be the case if there were no empirical law correlating the mental state with a physical state. It would be meaningless because no one would know how to verify it, but it would be significant for A if A actually had the feeling which he called " melancholy ". Meaningful statements comprise our knowledge, significant statements are descriptive of our experience. Meaning is communicable, significance is not. " A sees red " has meaning for everybody, " I feel noble " has significance for certain people.

When a statement is meaningful (belongs to empirical science) we assume, for all practical and scientific purposes, that when it is significant (when what it expresses is experienced) it has the same significance for everybody. On the other hand, when a statement does not belong to empirical science we assume that it may have a variable significance. Thus " I(A) see(s) red " is assumed to have the same significance (i.e. " seeing red " is the same experience) for different people ; " I feel noble " is assumed to have a variable significance (i.e. the experience may be of a different quality for different people).

V. Juhos would be partly right, when he says that the truth or falsity of a psychological statement " is independent of the behaviourist test ", if he meant that in so far as a statement is significant it need not be verified in the same way that it is verified in so far as it has meaning. For in the one case it can be determined to be true or false by the subject according as he does or does not have the feeling, in the other case it is determined to be true or false by others according as they do or do not observe the overt effects of the feeling. We may say that for the subject the statement is *significantly verifiable*.

Thus significant statements may be divided into two classes : those which have meaning, and those which have not. All significant statements, however, are significantly verifiable ; and a significant statement may be defined as one which is significantly verifiable. " I feel noble " is meaningless, but for

someone who has experienced a certain feeling which he calls " noble " it either is or is not the case that he has that feeling ; the statement is significantly verifiable (i.e. verifiable for him). But v. Juhos seems to hold that all significantly verifiable statements are meaningful, because he thinks that experiences are somehow communicable.

The foregoing applies only to psychological statements, of the form " I have a pain ", " A feels dizzy ", " I see red ". The word " significance " is not used in the sense that poetic phrases or expressions are said to be significant. These are not significantly verifiable.

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A SIMPLE METHOD OF TESTING TRUTH-FUNCTIONS

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IN applications of mathematical logic it is sometimes convenient to be able to discover rapidly whether a given truth-function of n elementary propositions is analytical (or tautological) or not. When n is greater than 2 or when the function is otherwise complicated the proof by the methods of *Principia Mathematica* may be long, tedious and difficult, and may demand considerable skill in handling the calculus of propositions. The object of this paper is to provide a method which is extremely rapid, and so simple that it demands no skill beyond the ability to apply mechanically a few quite simple rules.

The method consists in co-ordinating (in one-one fashion) a certain complex symbol with each of the n propositions in accordance with Rule 1 below, and then combining these symbols into a single symbol in accordance with Rules 2 to 4. Whether the function is analytical or not can then be seen at once in accordance with Rule 5.

Rule 1. Instead of using p, q, r, s, etc., as propositional variables we shall use $p_1, p_2, p_3, \ldots p_n$, where n is the number of different variables in the function to be tested. Rule 1 states: Represent the mth proposition by

$$(1^{2^{m-1}}\,0^{2^{m-1}})^{2^{n-m}}$$

where the indices (except those involving m or n) are merely repetitional indices, and m takes all values from 1 to n. This rule means, therefore: Write down '1' 2^{m-1} times and follow it by the same number of zeros. Then repeat the whole expression until you have 2^{n-m} of them, enclosing the whole expression in brackets. For the sake of a name for reference purposes such a complex sign (which will contain 2^n terms, each of which is a '1' or a '0') will be called a 2^n-termed 'vector.'

Example: Suppose we have a truth-function of three variables, p, q, and r (or p_1, p_2 and p_3). Then p will be represented by (10101010) which can also be written $((10)^4)$ or $((1010)^2)$, q by (11001100) or $((1100)^2)$, and r by (11110000) or $(1^4 0^4)$.

Rule 2. The negation of a proposition is represented by writing its vector with 0 substituted for 1 and 1 for 0.

Example: If $n=3$, $\sim p$ will be represented by $((0101)^2)$, and $\sim q$ by $((0011)^2)$, and so on.

Rule 3. Two 2^n-termed vectors may be combined in either of two ways, called 'addition' $(+)$ and 'multiplication' $(\times)$, to form a new 2^n-termed vector. In combining them each term of the one is combined with the corresponding term of the other in accordance with the following rules:

Rule 3a: $1+1=1$; $1+0=0+1=1$; $0+0=0$
Rule 3b: $1\times 1=1$; $1\times 0=0\times 1=0$; $0\times 0=0$

What is meant by 'corresponding terms' of two vectors is shown by the following examples in which the vectors are written one above the other with the result below:

$$\begin{array}{r} (1010) \\ +(1100) \\ \hline (1110) \end{array} \qquad \begin{array}{r} (1010) \\ \times(1100) \\ \hline (1000) \end{array}$$

representing $(1010)+(1100)$ and $(1010)\times(1100)$ respectively.

Rule 4. When, in the truth-function to be tested, two propositional variables are joined by the sign of logical addition their corresponding vectors are to be added ; and when they are joined by the sign of logical multiplication the vectors are to be multiplied. When two variables are joined by other signs the expressions are to be transformed (in accordance with the P.M. definitions) into equivalent expressions involving only negation, logical sum or logical product, and the above rules applied. (E.g. '$p \supset q$' is most conveniently transformed into '$\sim p \vee q$'). When such logical sums or products are themselves joined by the signs of logical addition or multiplication etc., the corresponding operations are to be performed on the resulting vectors. Beginning with the innermost brackets and working outwards this procedure is to be continued until the whole function to be tested is represented by a single vector.

To complete Rule 2 we must now add :

Rule 2a. The negation of a truth-function is represented by writing its vector with 0 substituted for 1 and 1 for 0.

Example : Since (1110) represents '$p \vee q$' when $n=2$, '$\sim(p \vee q)$' will be represented by (0001) when $n=2$.

Rule 5. If the single vector resulting from the application of the foregoing rules to the function to be tested contains no zeros the truth-function is analytical (or a tautology) otherwise it is not. If it contains only zeros it is a contradiction.

Theorem : Any two truth-functions which have the same vector are equivalent. For let F and G be any two functions having the same vector V. Then (writing N'V, for the negation of V in the sense of rule 2) $F \supset G$ will be represented by N'$V+V$, and $G \supset F$ will also be represented by N'$V+V$, but N'$V+V$ can contain no zeros because to every '1' of N'V there must correspond a '0' in V, and to every '0' in N'V a '1' in V, hence '$F \supset G . G \supset F$' must be analytical, and so we must have $F \equiv G$.

Examples : To illustrate the whole procedure we can test the following three truth-functions which appear as theorems in *Principia Mathematica* (the number after the asterisk is the

number of the theorem in P.M.). In the first example $n=2$, in the second $n=3$, and in the third $n=4$. The sign '=' is used in these examples to connect a given truth-function with its corresponding vector, and also to connect two vectors joined by an operation-sign with the resulting vector obtained by carrying out the operation.

$*(2.15)$ $\sim p \supset q . \supset . \sim q \supset p$

$$\begin{aligned} \sim p \supset q &= (1010)+(1100)=(1110) \\ \sim q \supset p &= (1100)+(1010)=(1110) \\ \sim p \supset q . \supset . \sim q \supset p &= (0001)+(1110)=(1111) \end{aligned}$$

$*(2.85)$ $pvq . \supset . pvr : \supset : p . v . q \supset r$

$$\begin{aligned} pvq &= (1010)^2+(1100)^2=(1110)^2 \quad . \quad . \quad (1) \\ pvr &= (1010)^2+(1^4 0^4)=(1^4 1010) \quad . \quad . \quad (2) \\ (1) \supset (2) &= (0001)^2+(1^4 1010)=(1^4 1011) \quad . \quad (3) \end{aligned}$$

$$\begin{aligned} q \supset r &= (0011)^2+(1^4 0^4)=(1^4 0011) \\ p . v . q \supset r &= (10)^4+(1^2 1^2 0^2 1^2)=(1^4 1011) \quad . \quad . \quad (4) \\ (3) \supset (4) &= (0^4 0100)+(1^4 1011)=(1^4 1^4) \end{aligned}$$

$*(3.47)$ $p \supset r . q \supset s . \supset : p . q . \supset . r . s$

$$\begin{aligned} p \supset r &= (0101)^4+(1^4 0^4)^2=(1^4 0101)^2 \quad . \quad . \quad (1) \\ q \supset s &= (0011)^4+(1^8 0^8)=(1^8 0^2 1^2 0^2 1^2) \quad . \quad (2) \\ (1).(2) &= (1^4 0101)^2 \times (1^8 (0^2 1^2)^2) \\ &\qquad = (1^4 (01)^2 0^2 1^2 0^2 01) \quad (3) \\ p . q &= (1010)^4 \times (1100)^4=(1000)^4 \quad . \quad . \quad (4) \\ r . s &= (1^4 0^4)^2 \times (1^8 0^8)=(1^4 0^4 0^8) . \quad . \quad . \quad (5) \\ (4) \supset (5) &= (0111)^4+(1^4 0^4 0^8)=(1^4 (0111)^3) \quad . \quad (6) \\ (3) \supset (6) &= (0^4 (10)^2 1^2 0^2 1^2 10)+(1^4 (0111)^3) \\ &= (1^{16}) \end{aligned}$$

The method is based, and depends for its validity upon the truth-value tables. It represents, in fact, a generalization of the truth-tables for n propositions. Each so-called vector represents

a column of the corresponding truth-table (according to the value of n), with '1' instead of 'T,' and '0' in the place of 'F.' But in drawing up the truth-table it was necessary to distribute these two signs in a special way, namely in such a way as would make the vectors corresponding to p, q, r, etc., conform to the requirements which enable Rule 1 to be formulated for any value of n.